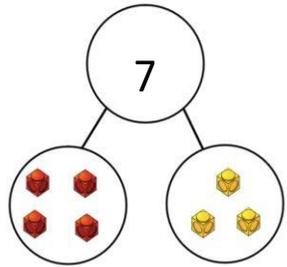




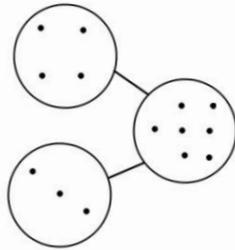
Addition &
SUBTRACTION

Part-Whole Model



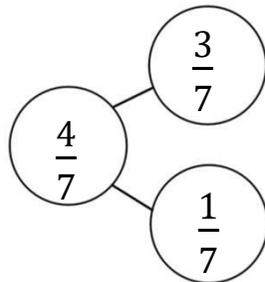
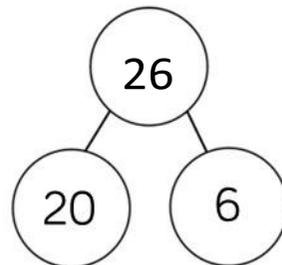
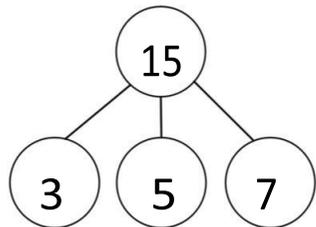
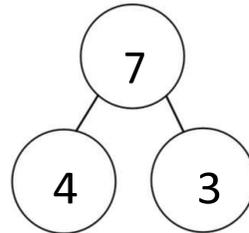
$$7 = 4 + 3$$

$$7 = 3 + 4$$



$$7 - 3 = 4$$

$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total. When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

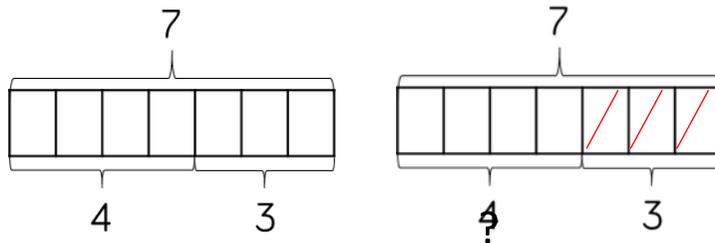
In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)

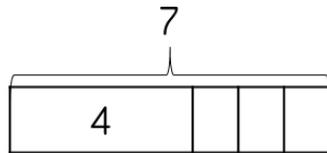
Concrete



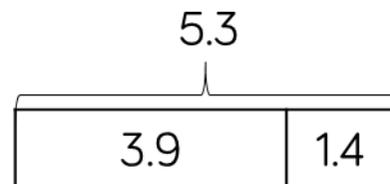
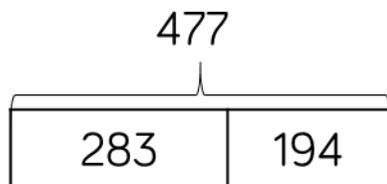
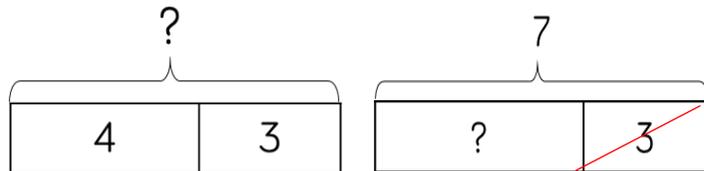
Discrete



Combination



Continuous



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

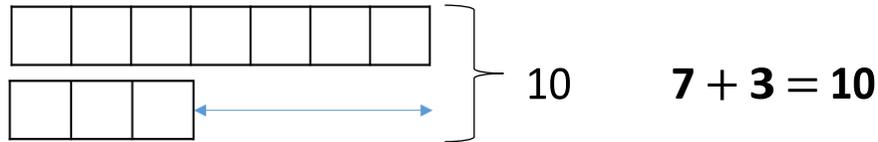
Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number

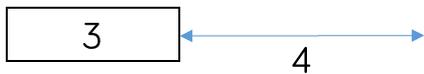
Bar Model (multiple)

Discrete

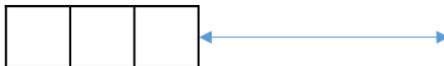
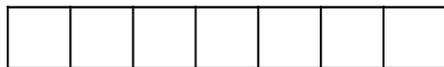


$$7 - 3 = 4$$

Continuous



$$7 - 3 = 4$$

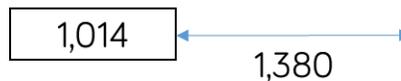
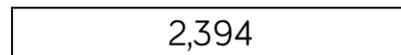


Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

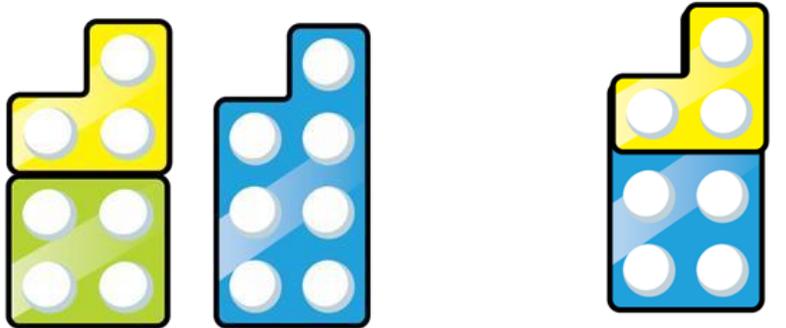
Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.



$$2,394 - 1,014 = 1,380$$

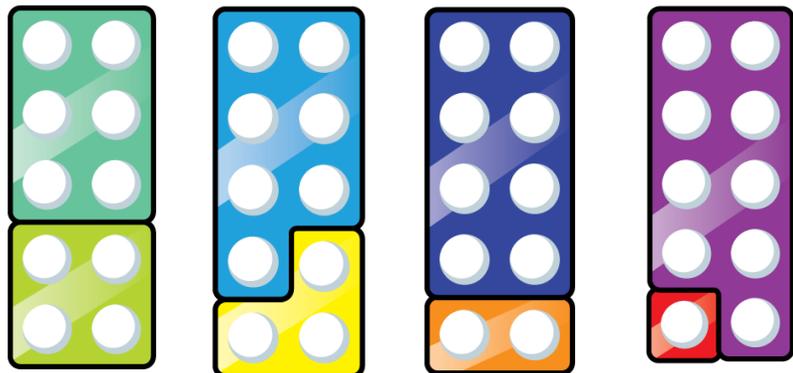
Number Shapes



$7 = 4 + 3$

$7 = 3 + 4$

$7 - 3 = 4$



$6+4$

$7+3$

$8+2$

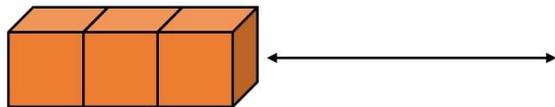
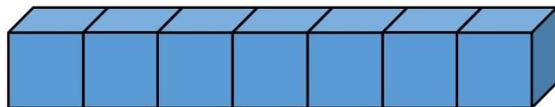
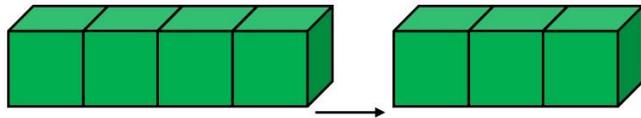
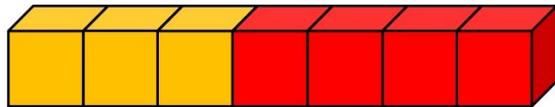
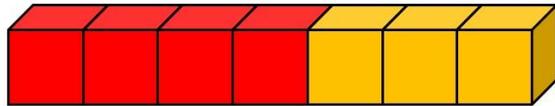
$9+1$

Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

Cubes



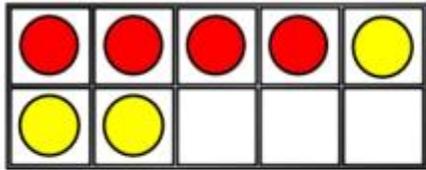
Benefits

When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Ten Frames (within 10)



$$4 + 3 = 7$$

$$3 + 4 = 7$$

$$7 - 3 = 4$$

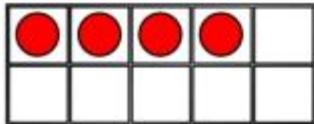
$$7 - 4 = 3$$

4 is a part.

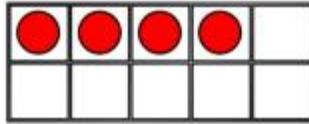
3 is a part.

7 is the whole.

First

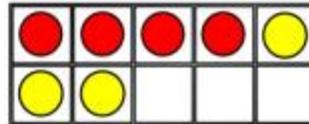


Then

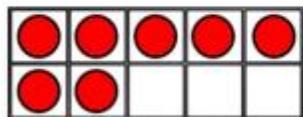


$$4 + 3 = 7$$

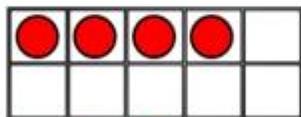
Now



First

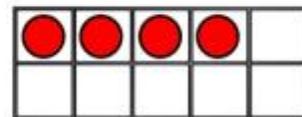


Then



$$7 - 3 = 4$$

Now



Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

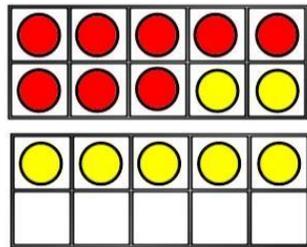
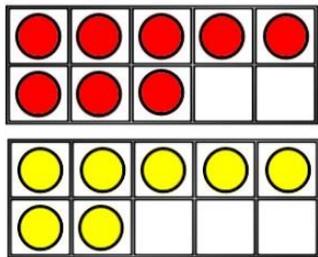
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts.

Using these structures, the ten frame can enable children to find all the number bonds for a number.

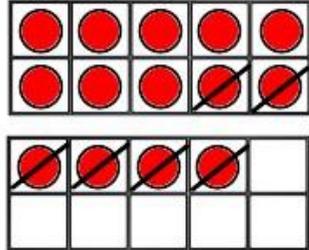
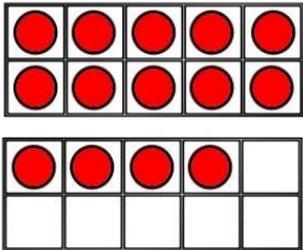
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



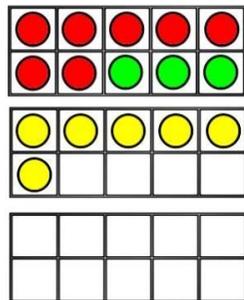
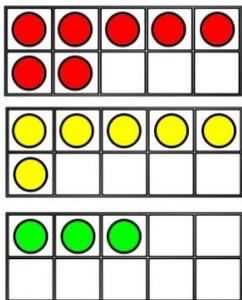
$$8 + 7 = 15$$

2



$$14 - 6 = 8$$

4



$$7 + 6 + 3 = 16$$

10

Benefits

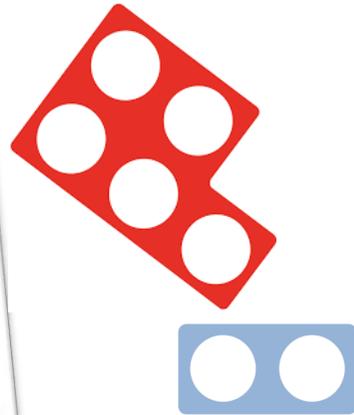
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames or rekenrek maths counting frame.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

Addition

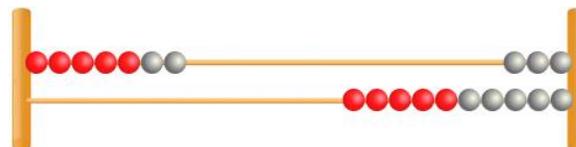
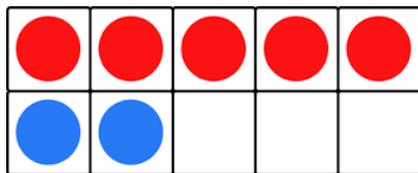
Skill: Adds on numbers to make one number into another

Early Years



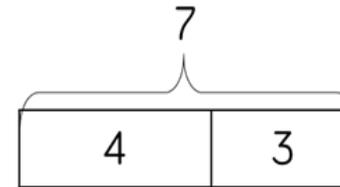
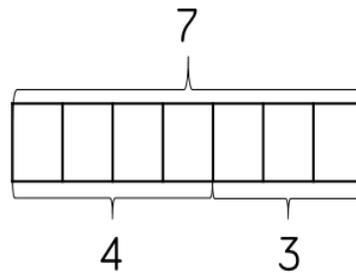
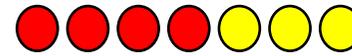
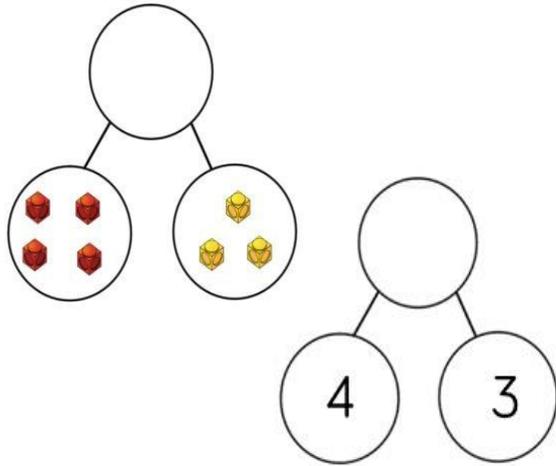
Use **direct modelling** and **count all** strategies with objects to solve simple addition problems - augmentation (adding to a quantity).

The rekenrek and tens frame helps pupils to **subitize up to and beyond 10**.

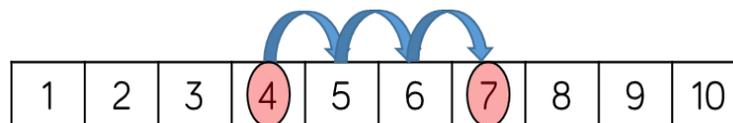
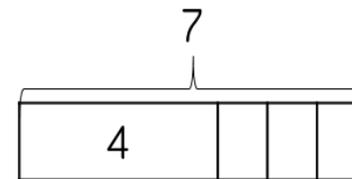
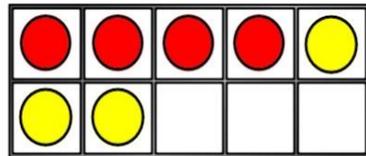
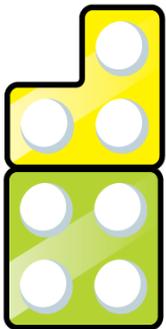


Skill: Add 1-digit numbers within 10

Early Years/Year: 1



$$4 + 3 = 7$$



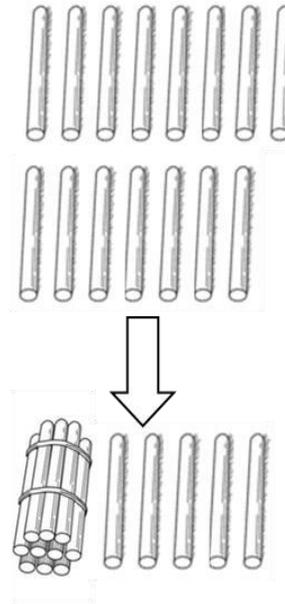
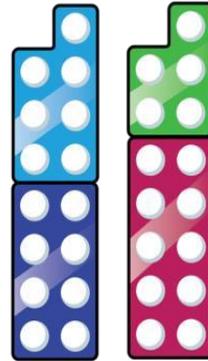
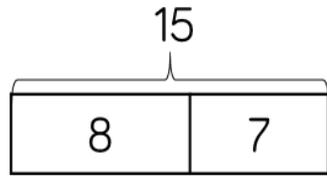
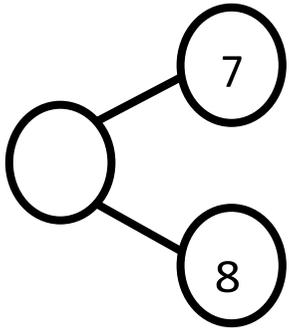
When adding numbers to 10, children can explore aggregation (combining two or more parts to make a whole) and augmentation (adding to a quantity).

The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation.

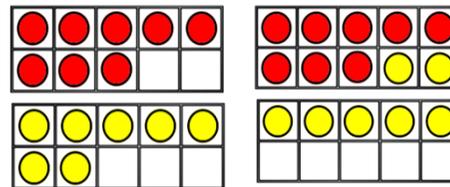
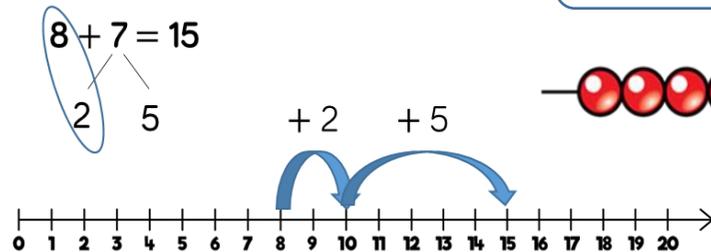
The combination bar model, ten frame, bead string and number track all support augmentation.

Skill: Add 1 and 2 -digit numbers to 20

Year: 1/2



8 + 7 = 15

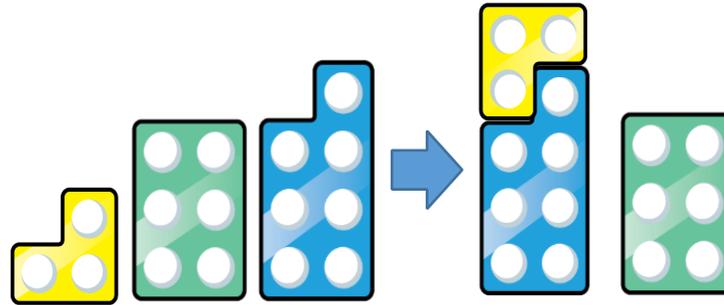
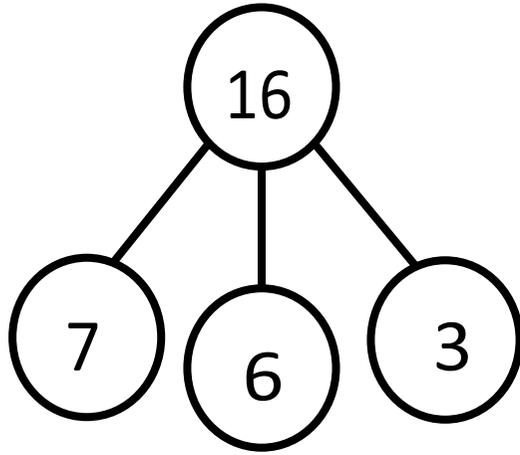


8 + 7 = 15
2 5

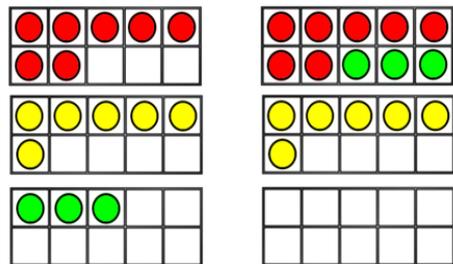
When adding one - digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. In Year 1, this is only done just by counting on. From Year 2, use different manipulatives can be used to represent this exchange alongside number lines to support children in understanding how to partition their jumps.

Skill: Add three 1 -digit numbers

Year: 2

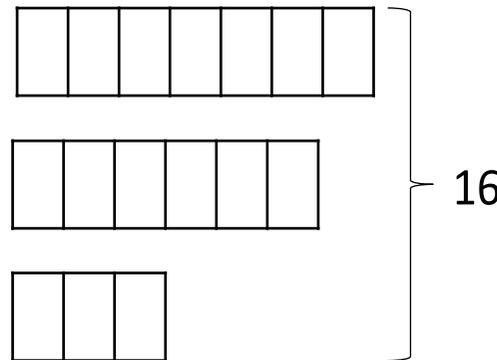


$$7 + 6 + 3 = 16$$



$$7 + 6 + 3 = 16$$

10

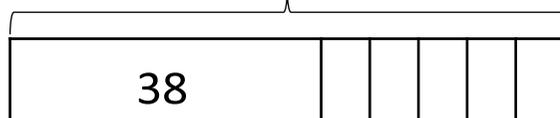
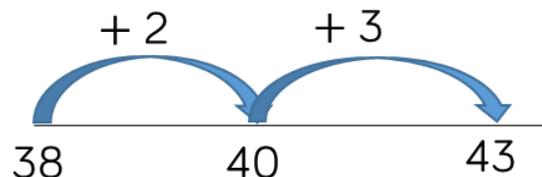
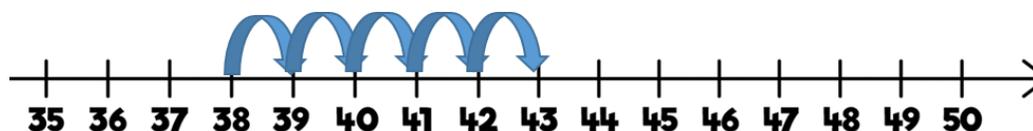
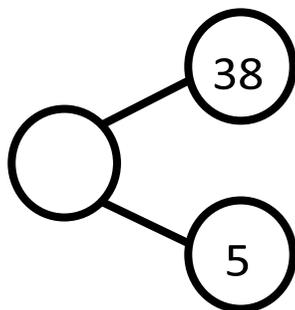


When adding three 1 -digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity.

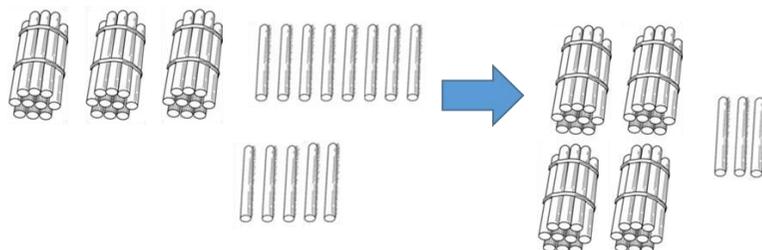
Manipulatives that highlight number bonds to 10 are effective when adding three 1 -digit numbers.

Skill: Add 1-digit and 2-digit numbers to 100

Year: 2/3



$$38 + 5 = 43$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

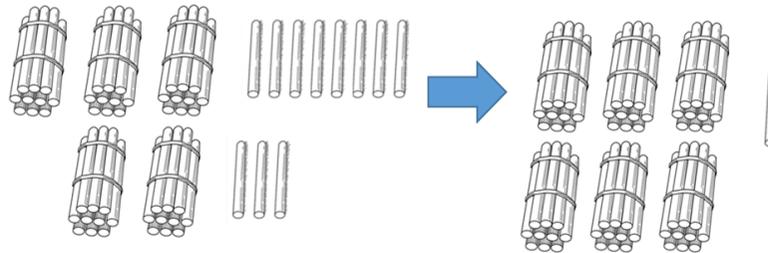
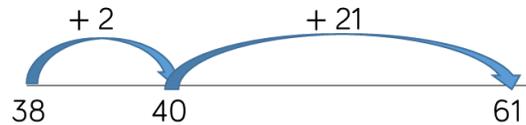
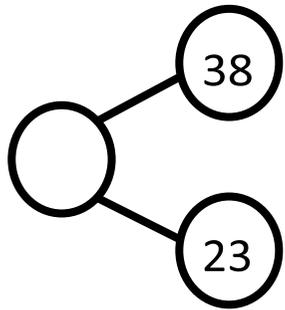
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$.

Hundred squares and straws can support children to find the number bond to 10

Skill: Add two 2-digit numbers to 100

Year: 2/3



?	
38	23

$$38 + 23 = 61$$

Tens	Ones

$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ \hline 1 \end{array}$$

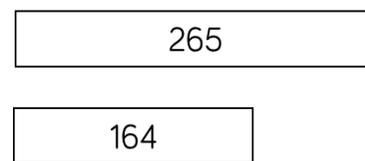
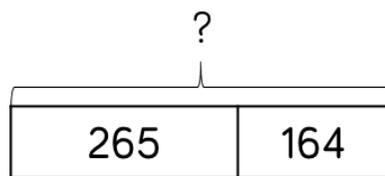
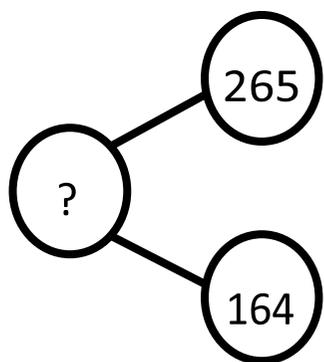
Tens	Ones
10 10 10	1 1 1 1 1 1 1 1
10 10	1 1 1
10	

Children can use a blank number line and other representations to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

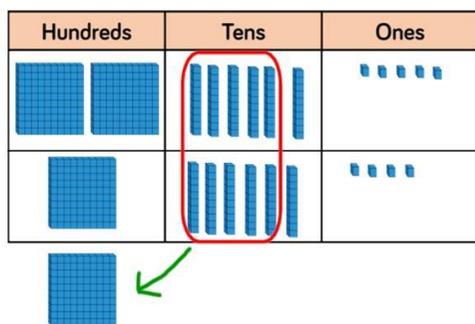
From Year 3, encourage children to use the formal column method when calculating or base 10 or place value counters.

Skill: Add numbers with up to 3 digits

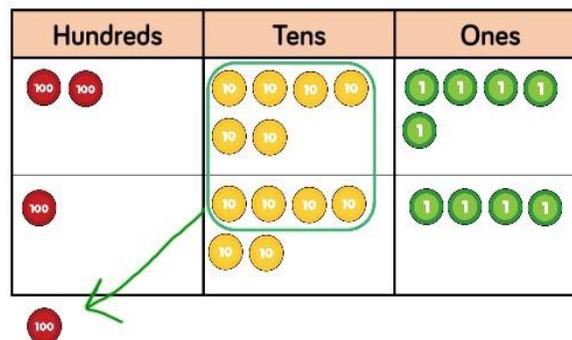
Year: 3



$$265 + 164 = 429$$



$$\begin{array}{r} 1 \\ 265 \\ + 164 \\ \hline 429 \end{array}$$



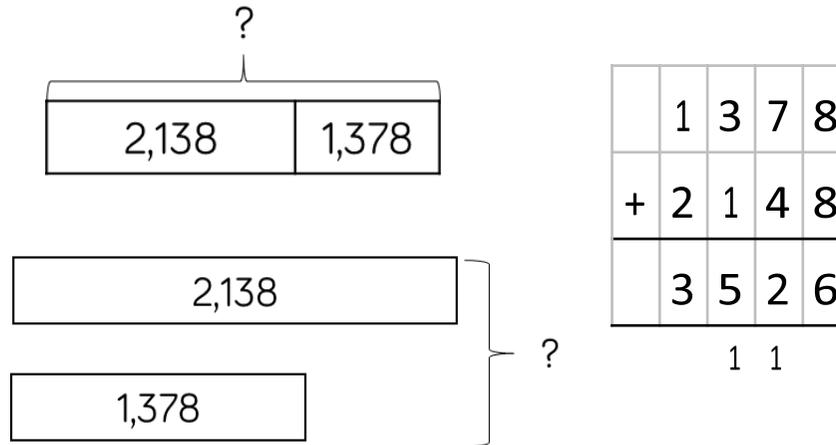
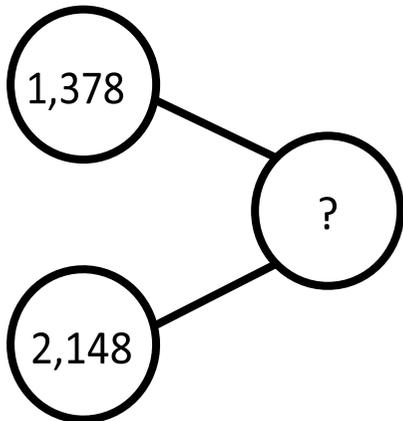
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

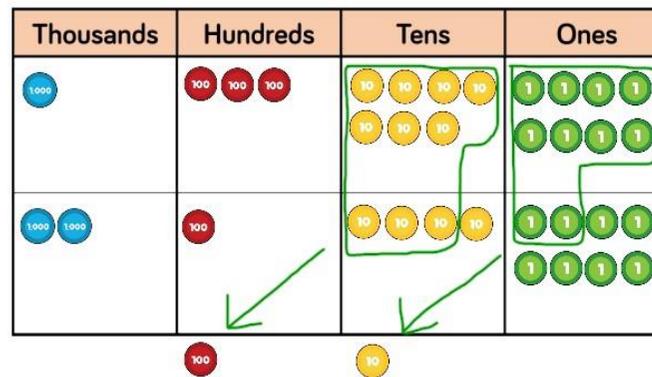
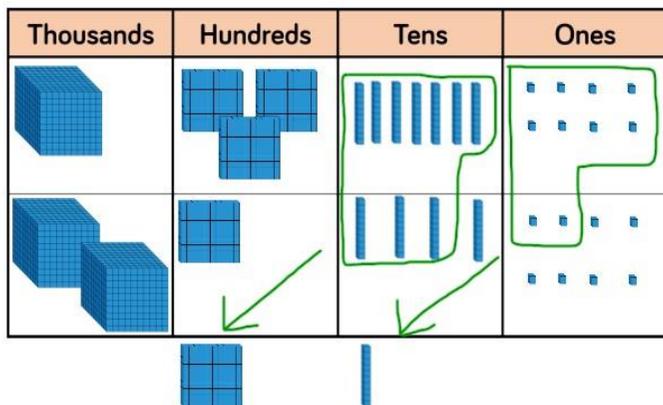
Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with up to 4 digits

Year: 4



$$1,378 + 2,148 = 3,526$$



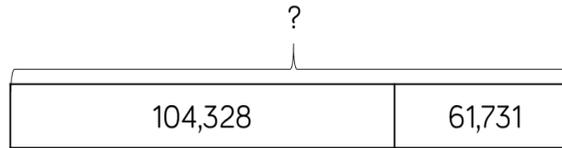
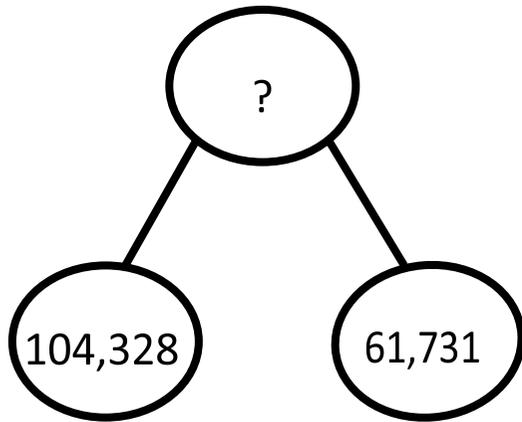
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with more than 4 digits

Year: 5/6



$$104,328 + 61,731 = 166,059$$

HTh	TTh	Th	H	T	O
100,000		1,000 1,000 1,000 1,000	100 100 100	10 10	1 1 1 1 1
	10,000 10,000 10,000	1,000	100 100 100 100 100	10 10 10	1

1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9

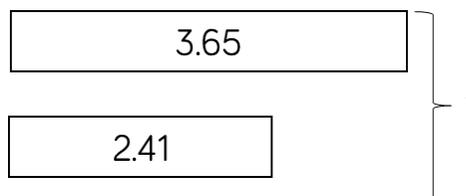
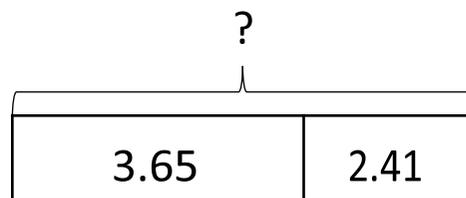
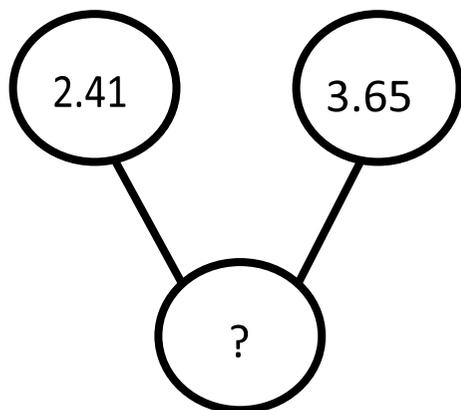
1

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

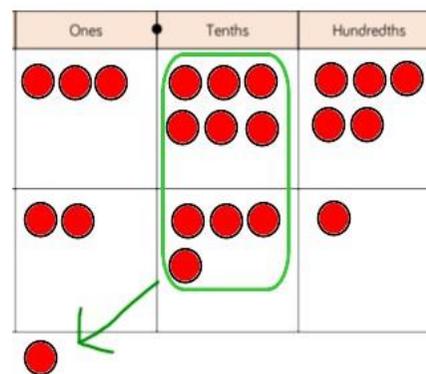
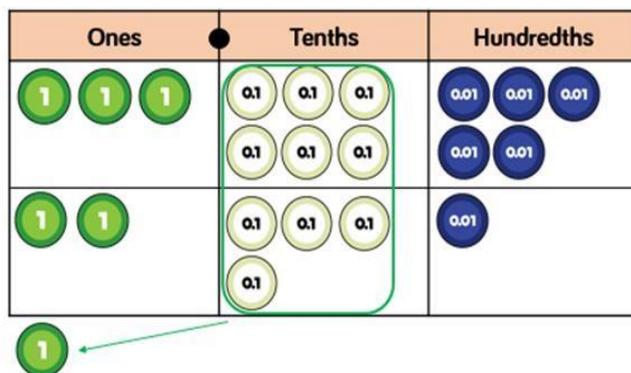
Skill: Add with up to 3 decimal places

Year: 5



$$\begin{array}{r}
 3.65 \\
 + 2.41 \\
 \hline
 6.06 \\
 1
 \end{array}$$

$3.65 + 2.41 = 6.06$



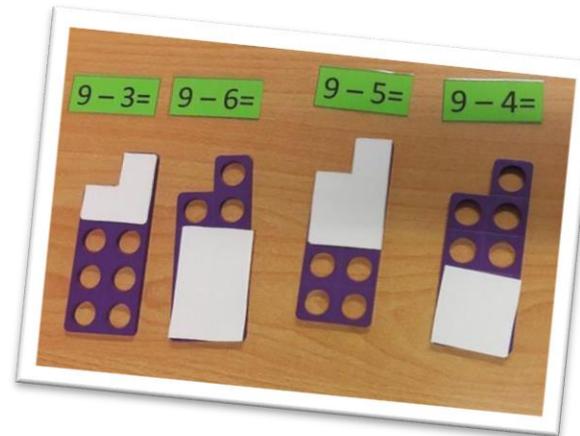
Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

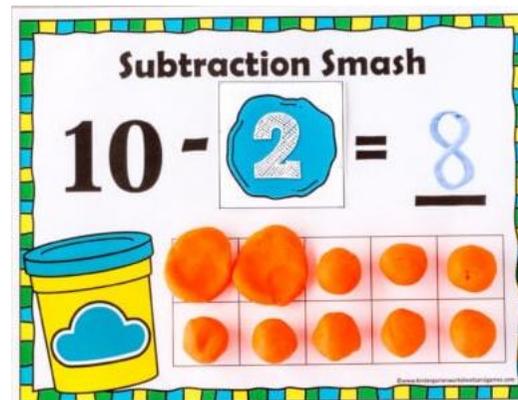
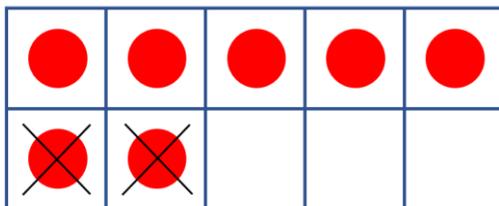
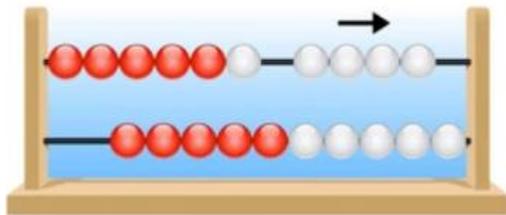
Subtraction

Skill: Separating to decrease the number in a set

Early Years



$$10 - 4 = 6$$

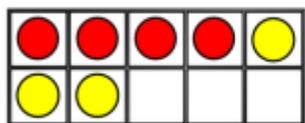
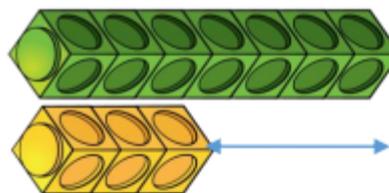
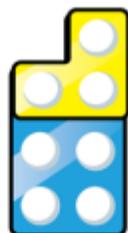
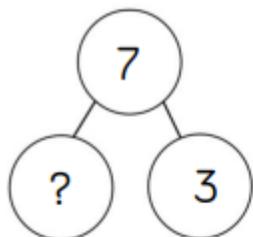


Use **direct modelling** with objects and the action of separating or taking away to decrease the number in a set.

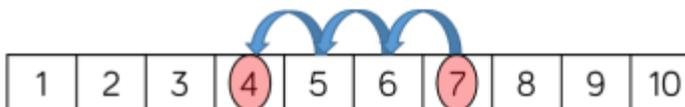
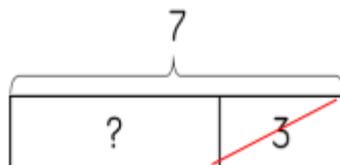
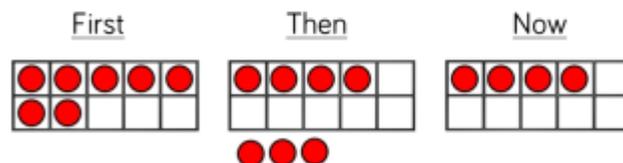
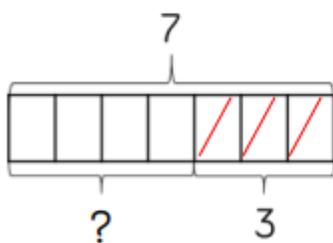
The rekenrek and tens frame helps pupils to **subitize up to and beyond 10**.

Skill: Subtract 1-digit numbers within 10

Early Years/Year: 1



$$7 - 3 = 4$$



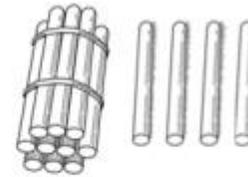
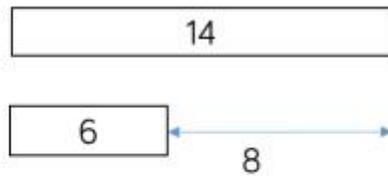
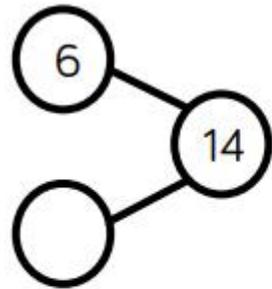
Part-whole models, bar models, ten frames and number shapes support partitioning.

Ten frames, number tracks, single bar models and bead strings support reduction.

Cubes and bar models with two bars can support finding the difference.

Skill: Subtract 1 and 2-digit numbers to 20

Year: 1/2

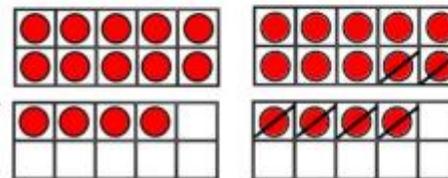
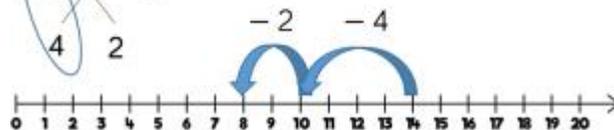


$$14 - 6 = 8$$



$$14 - 6 = 8$$

A diagram showing the number 14 with a bracket around it. A diagonal line splits it into 4 and 10. The 4 is circled, and the 10 is split into 2 and 8.



$$14 - 6 = 8$$

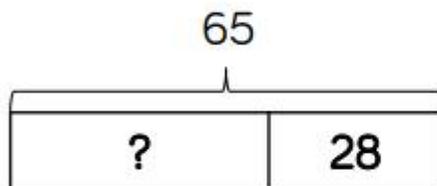
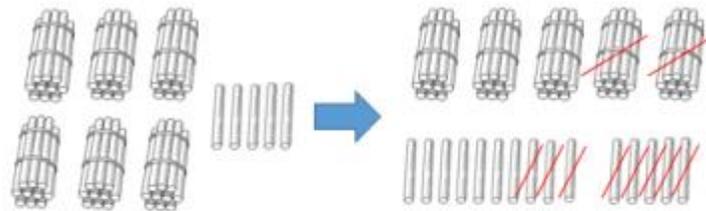
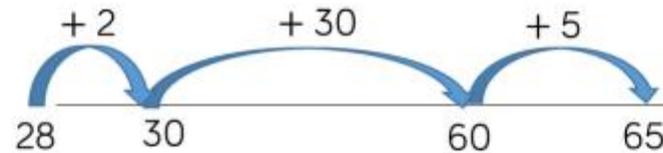
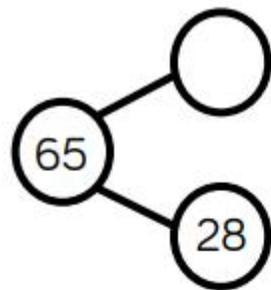
A diagram showing the number 14 with a bracket around it. A diagonal line splits it into 4 and 10. The 4 is circled, and the 10 is split into 2 and 8.

In Early Years and Year 1, subtracting one-digit numbers that cross 10, is done by counting back, using objects, number tracks and number lines.

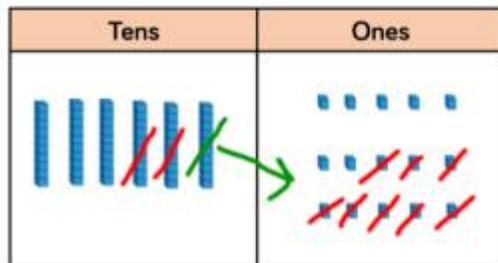
From Year 2, children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

Skill: Subtract 1 and 2-digit numbers to 100

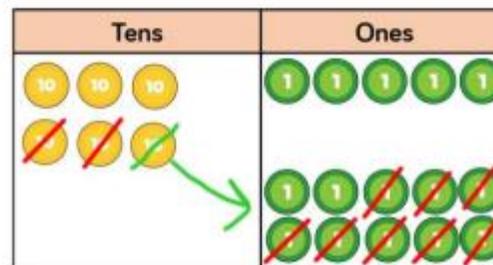
Year: 2/3



$$65 - 28 = 37$$



$$\begin{array}{r} 5 \ 1 \\ 65 \\ - 28 \\ \hline 37 \end{array}$$



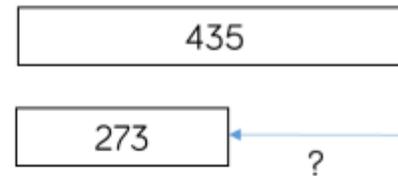
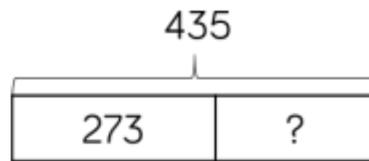
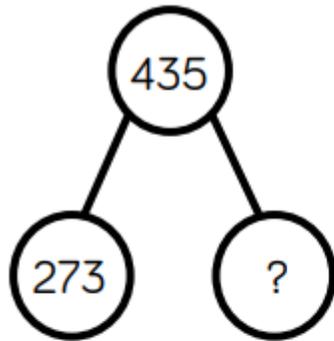
Children can also use a blank number line to count back to find the difference.

Encourage them to jump to multiples of 10 to become more efficient.

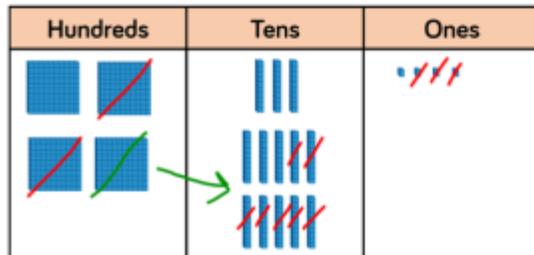
From Year 3, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters.

Skill: Subtract numbers with up to 3 digits

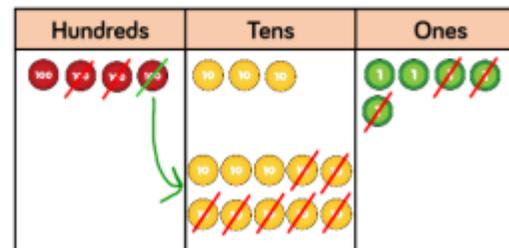
Year: 3



$$435 - 273 = 162$$



$$\begin{array}{r} 3 \quad 1 \\ 435 \\ - 273 \\ \hline 162 \end{array}$$



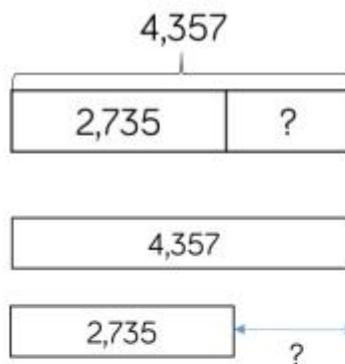
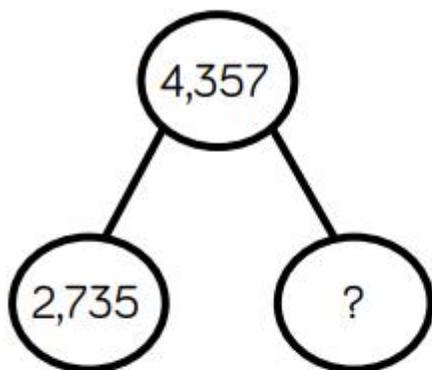
Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

Plain counters on a place value grid can also be used to support learning

Skill: Subtract numbers with up to 4 digits

Year: 4



$$\begin{array}{r} 3 \ 1 \\ 4357 \\ - 2735 \\ \hline 1622 \end{array}$$

$$4,357 - 2,735 = 1,622$$

Thousands	Hundreds	Tens	Ones

Thousands	Hundreds	Tens	Ones

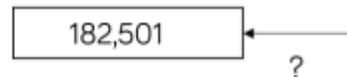
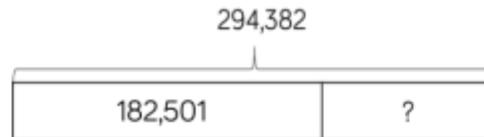
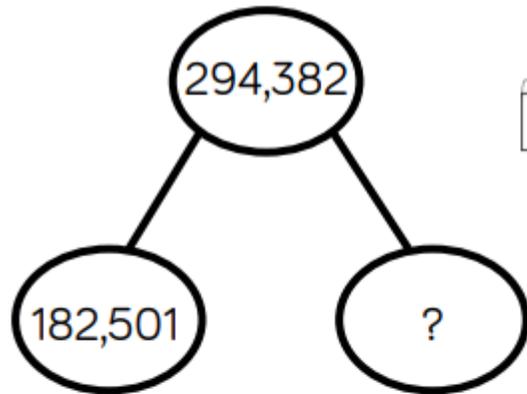
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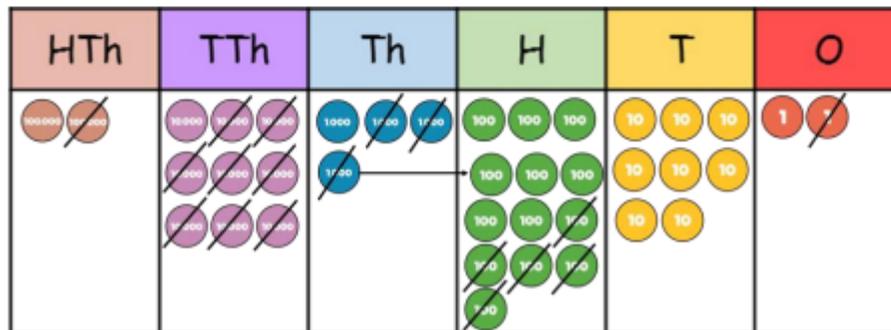
Plain counters on a place value grid can also be used to support learning.

Skill: Subtract numbers with more than 4 digits

Year: 5/6



$$294,382 - 182,501 = 111,881$$



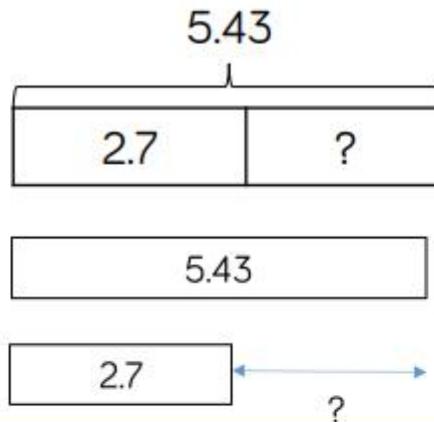
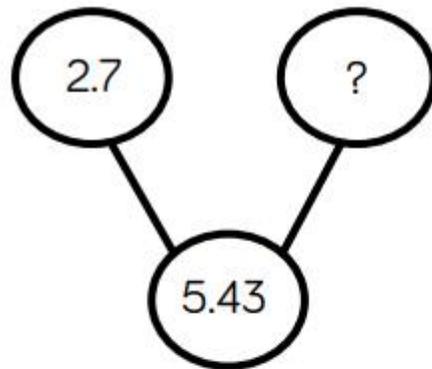
	2	9	3	1 ₃	8	2
-	1	8	2	5	0	1
	1	1	1	8	8	1

Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently

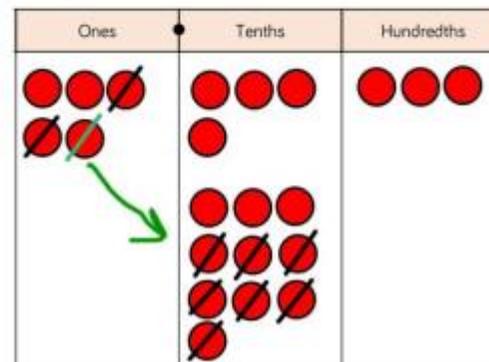
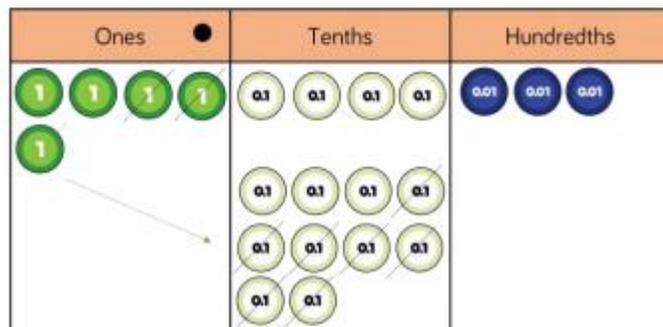
Skill: Subtract with up to 3 decimal places

Year: 5/6



$$\begin{array}{r} 4 \ 1 \\ \cancel{5}.43 \\ - 2.7 \\ \hline 2.73 \end{array}$$

$$5.43 - 2.7 = 2.73$$



Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.